ME 6135: Advanced Aerodynamics

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Lecture-13

30/11/2024

2D flows
Oblique Shock Waves

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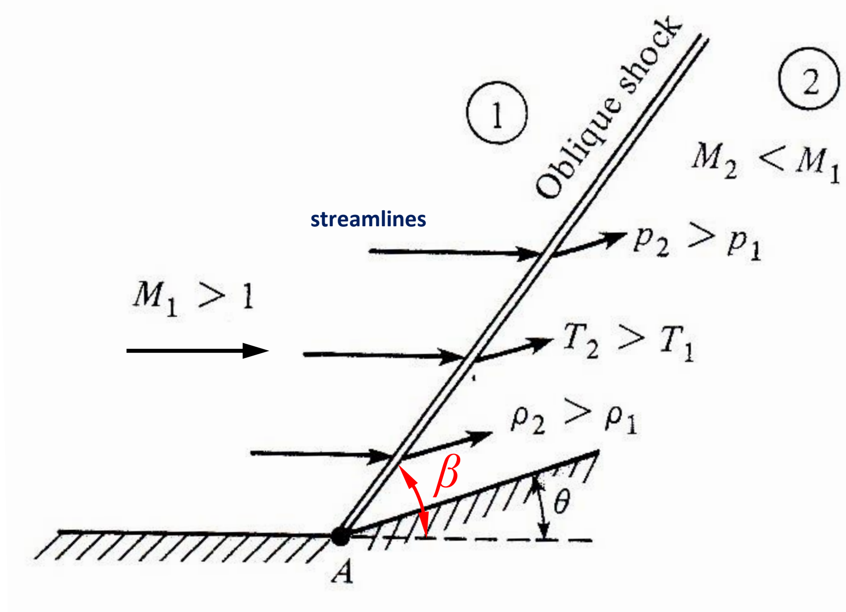
Oblique Shock Waves

Oblique shock waves usually occur when supersonic flow (M > 1.0) is "**turned into itself**" as shown in Fig. 4.4(a).

At point A, the surface is deflected upward through an angle ***θ*** (***deflection angle***). Consequently, the flow streamlines are deflected upward (to follow the geometric change), toward the main bulk of the flow above the surface.

This change in flow direction takes place across a shock wave (at some angle known as ***shock wave angle, β***) which is in oblique orientation to the free-stream direction.

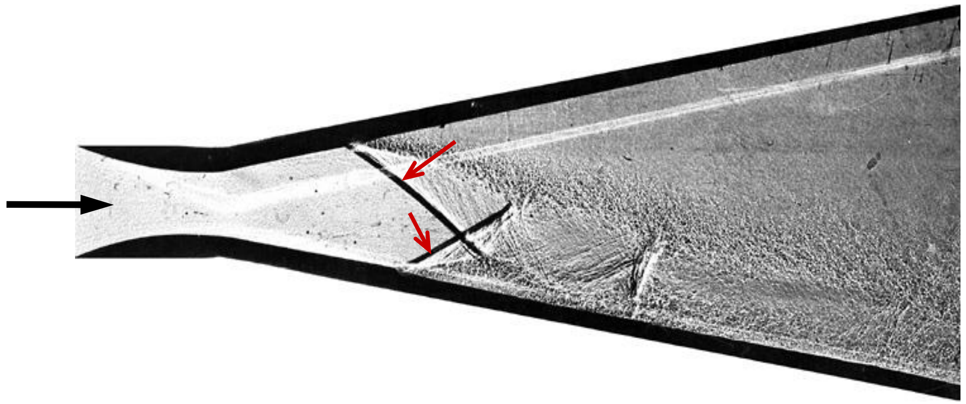


(a) Concave corner

Figure 4.4 | Supersonic flow over a corner.

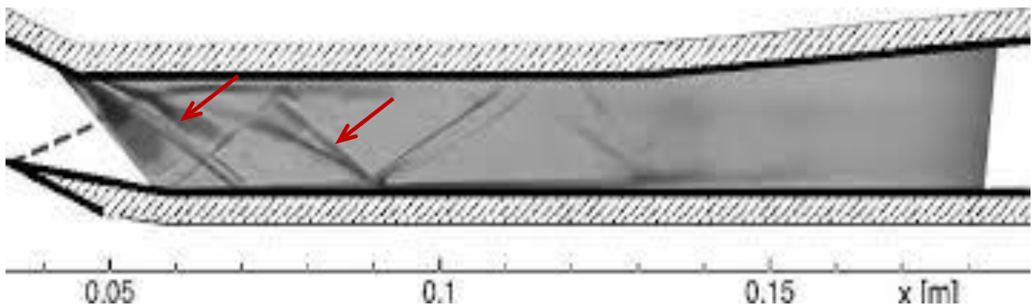
Across the oblique shock wave, the Mach number decreases, and the pressure, temperature, and density increase.

Oblique Shock Waves

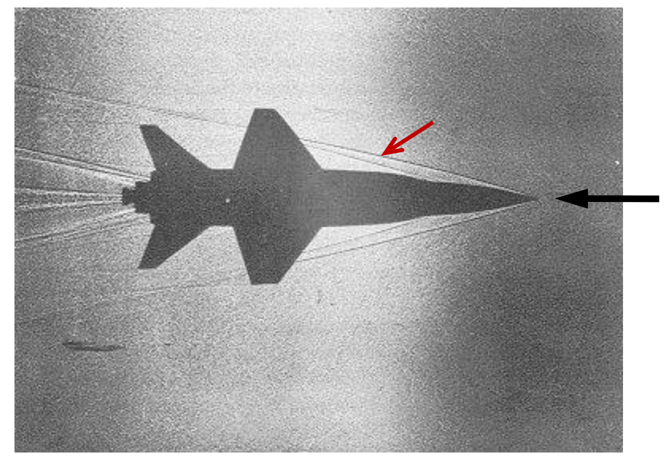


Asymmetric oblique shock wave in a rocket nozzle

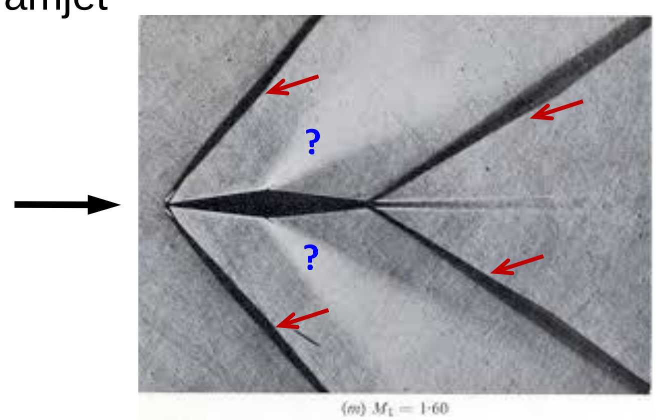
Oblique shock wave in a diverging passage



Oblique shock wave in an inlet isolator in Scramjet



F-16 in supersonic speed

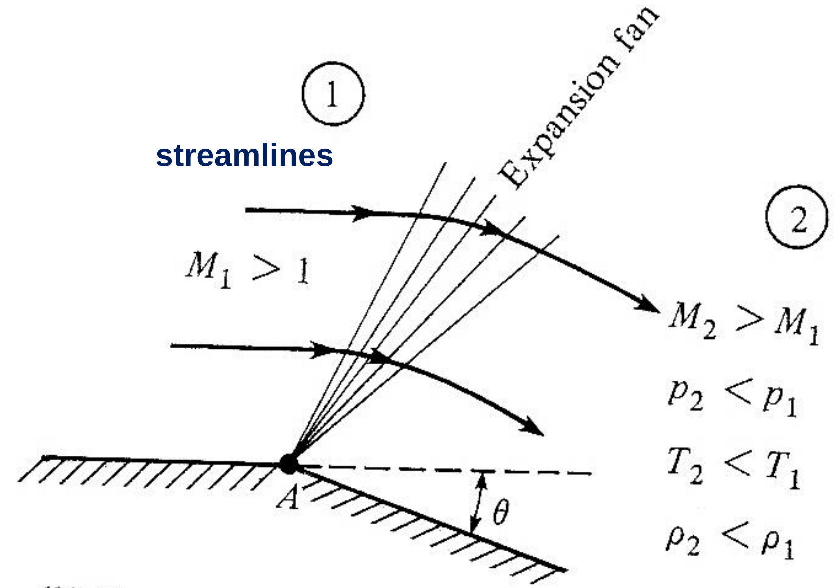


Diamond shaped airfoil in *M* >1

Expansion Waves

When a supersonic flow is "**turned away from itself**" as shown in Fig. (b), an ***expansion wave*** is formed.

Here, the surface is **deflected downward through an angle *θ*.** Consequently the flow streamlines are deflected downward, away from the main bulk of flow above the surface. This change in flow direction takes place across an **expansion wave (expansion fan)**, centered at point A. Away from the surface, this oblique expansion fans out, as shown in Fig. (b).



(b) Convex corner

The flow streamlines are smoothly curved through an expansion fan until they became parallel to the wall behind point A. Hence, the flow behind the expansion wave is also uniform and parallel, in the direction of θ.

Across the expansion wave, the Mach number increases and the pressure, temperature, and density decreases.

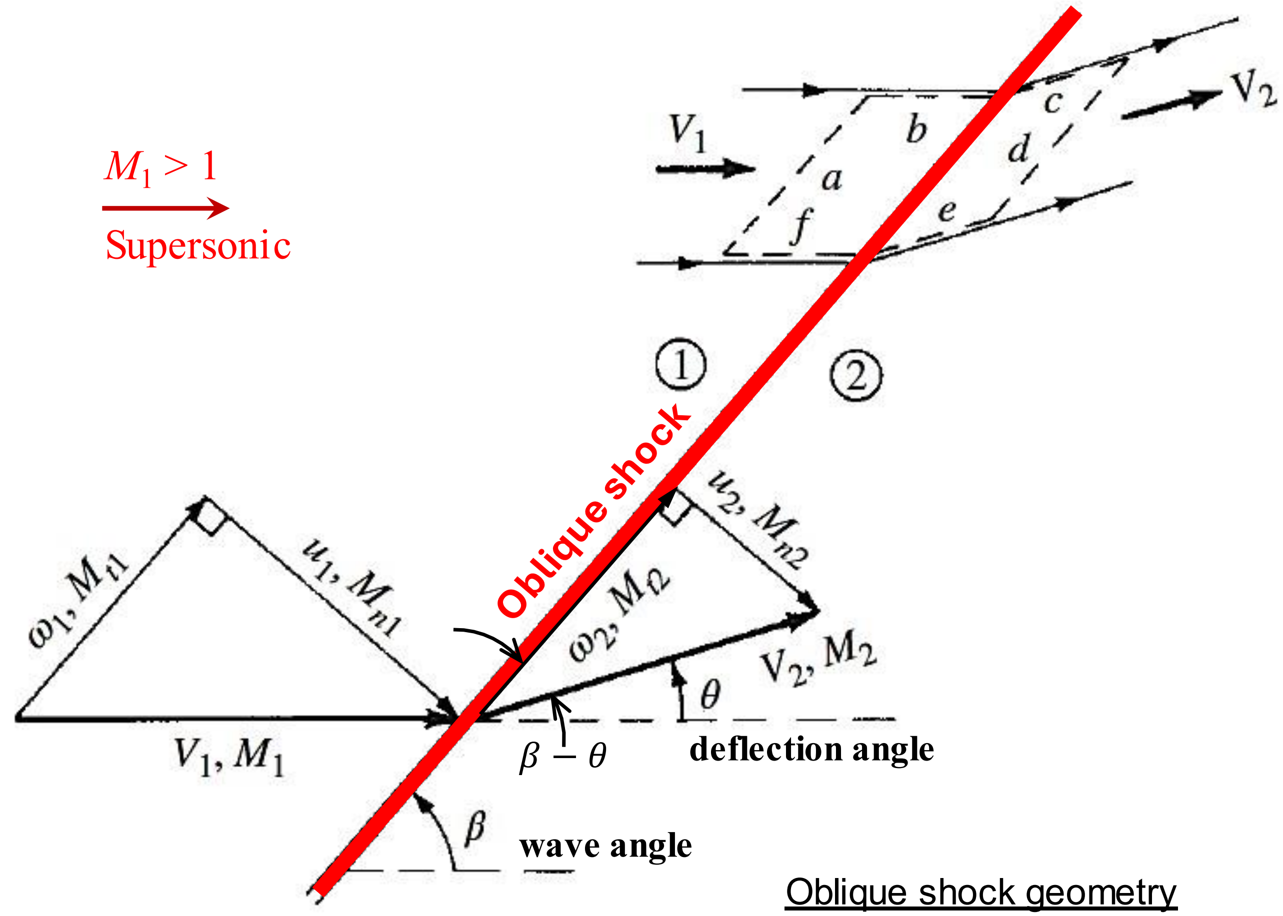
Oblique shock and expansion waves are inherently two-dimensional (2D) in nature, in contrast to the one-dimensional normal shock waves. That is, the flow properties are functions of *x* and *y*-coordinates.

Oblique Shock Waves

The geometry of flow through an oblique shock wave is given in Figure.

The velocity upstream of the shock is V_1 and is horizontal. The corresponding Mach number is M_1 which is supersonic ($M > 1$).

The oblique shock makes a **wave angle β with respect to V_1.** Behind the shock, the flow is deflected toward the shock by the **flow deflection angle θ** (angle of physical geometry with respect to the upstream free stream direction).



Oblique shock geometry

The velocity and Mach number behind the shock wave are V_2 and M_2, respectively.

The components of V_1 perpendicular and parallel, respectively, to the shock are u_1 and w_1; analogous components of V_2 are u_2 and w_2, as shown in Figure.

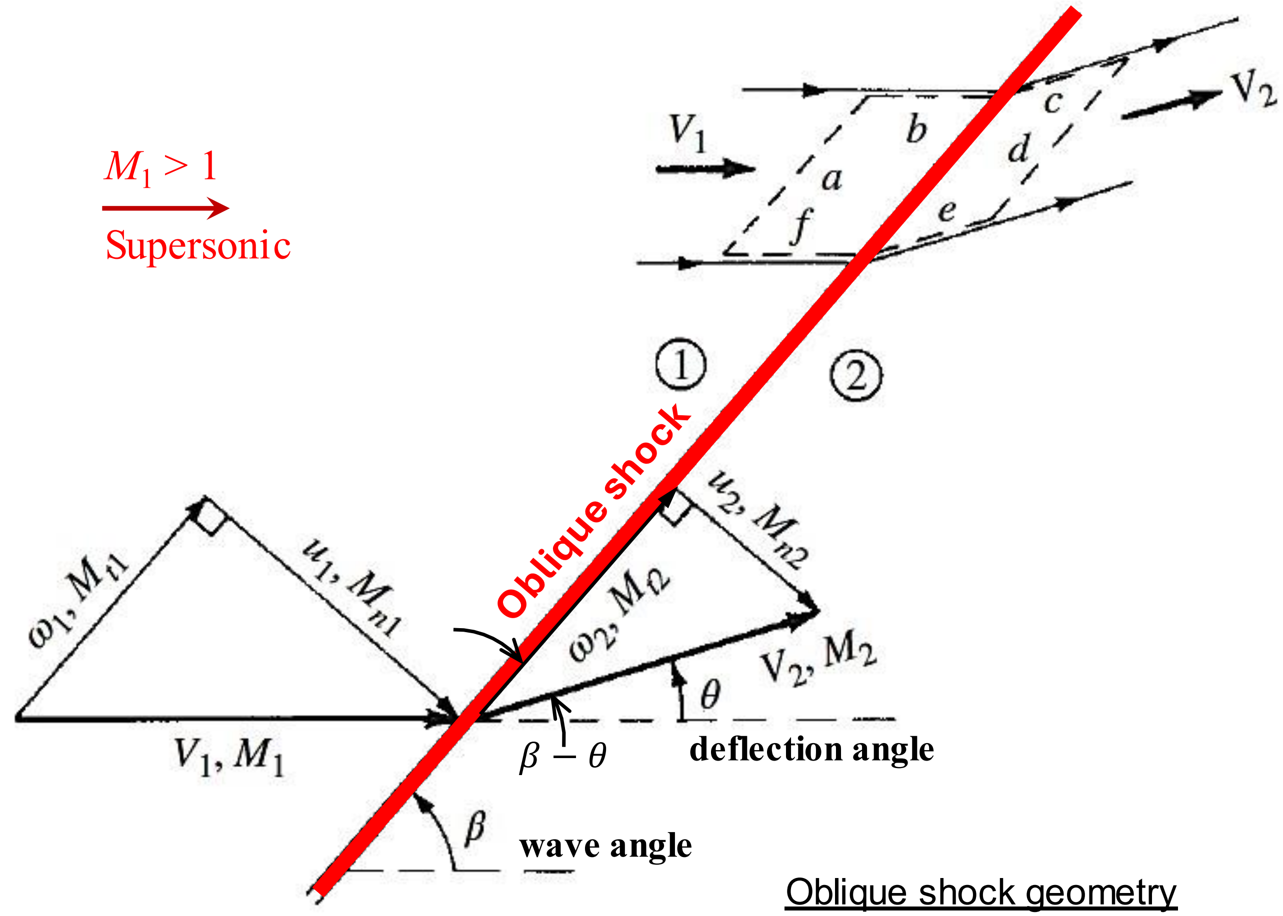
Therefore, it can be considered the normal and tangential Mach numbers ahead of the shock to be M_{n1} and M_{t1}, respectively; similarly, we have M_{n2} and M_{t2} behind the shock.

Oblique Shock Waves

Consider the control volume (abcdef) drawn between two streamlines through an oblique shock. Faces *a* and *d* are parallel to the shock wave (oblique).
The continuity equation applied to the control volume gives (**normal components**) -

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2 \;\Rightarrow\; \rho_1 u_1 = \rho_2 u_2$$

$$A_1 = A_2 = \text{area of faces } a \text{ and } d$$



Oblique shock geometry

Considering steady flow with no body forces, the integral form of momentum equation:

$$\oiint_S (\rho \mathbf{V}\cdot d\mathbf{S})\mathbf{V} + \iiint_{\forall} \frac{\partial(\rho \mathbf{V})}{\partial t}\, d\forall = \iiint_{\forall} \rho \mathbf{f}\, d\forall - \oiint_S p\, d\mathbf{S}$$

Integral form of momentum equation

tangential components:

$$(-\rho_1 u_1 A_1) w_1 + (\rho_2 u_2 A_2) w_2 = 0 \;\Longleftarrow$$

$$\boxed{\Rightarrow w_1 = w_2}$$

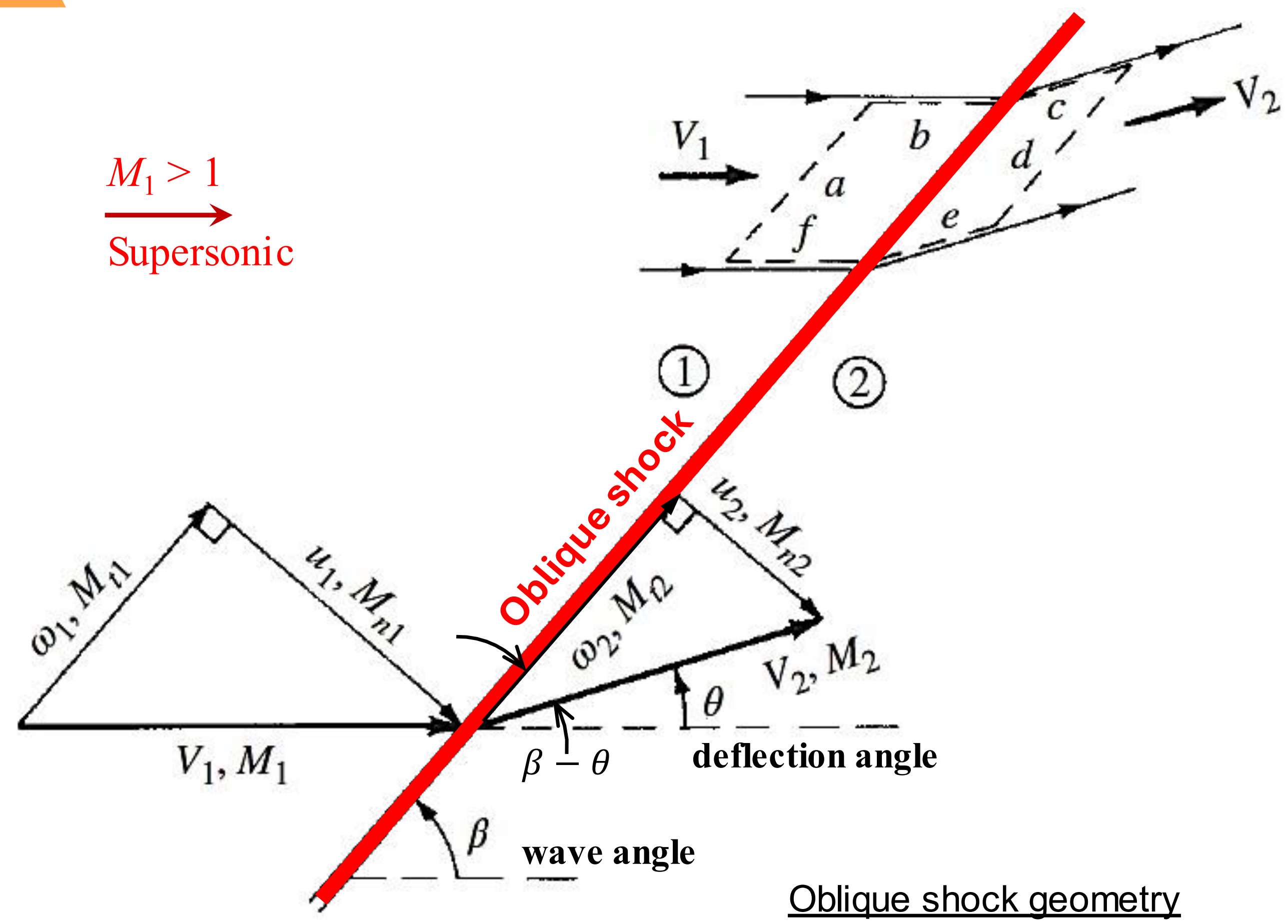
tangential component of *p*d**S** is zero on faces *a* and *d*. The components on *b* cancel those on *f*; similarly with faces *c* and *e*.

The tangential component of the flow velocity is preserved across an oblique shock wave.

normal component in momentum equation:

$$(-\rho_1 u_1 A_1)u_1 + (\rho_2 u_2 A_2)u_2 = -(-p_1 A_1 + p_2 A_2)$$

$$\Rightarrow p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$



Oblique shock geometry

Next, the integral form of the **energy equation** applied to the control volume for a steady adiabatic flow with no body force yields:

$$\iiint_{\mathcal{V}} \dot{q}\rho\, d\mathcal{V} - \oiint_{S} p\mathbf{V}\cdot d\mathbf{S} + \iiint_{\mathcal{V}} \rho(\mathbf{f}\cdot\mathbf{V})\, d\mathcal{V} = \iiint_{\mathcal{V}} \frac{\partial}{\partial t}\left[\rho\left(e + \frac{V^2}{2}\right)\right] d\mathcal{V} + \oiint_{S} \rho\left(e + \frac{V^2}{2}\right)\mathbf{V}\cdot d\mathbf{S}$$

Integral form of energy equation

$$-(-p_1\rho_1 u_1^2 + p_2\rho_2 u_2^2) = -\rho_1\left(e_1 + \frac{V_1^2}{2}\right)u_1 + \rho_2\left(e_2 + \frac{V_2^2}{2}\right)u_2$$

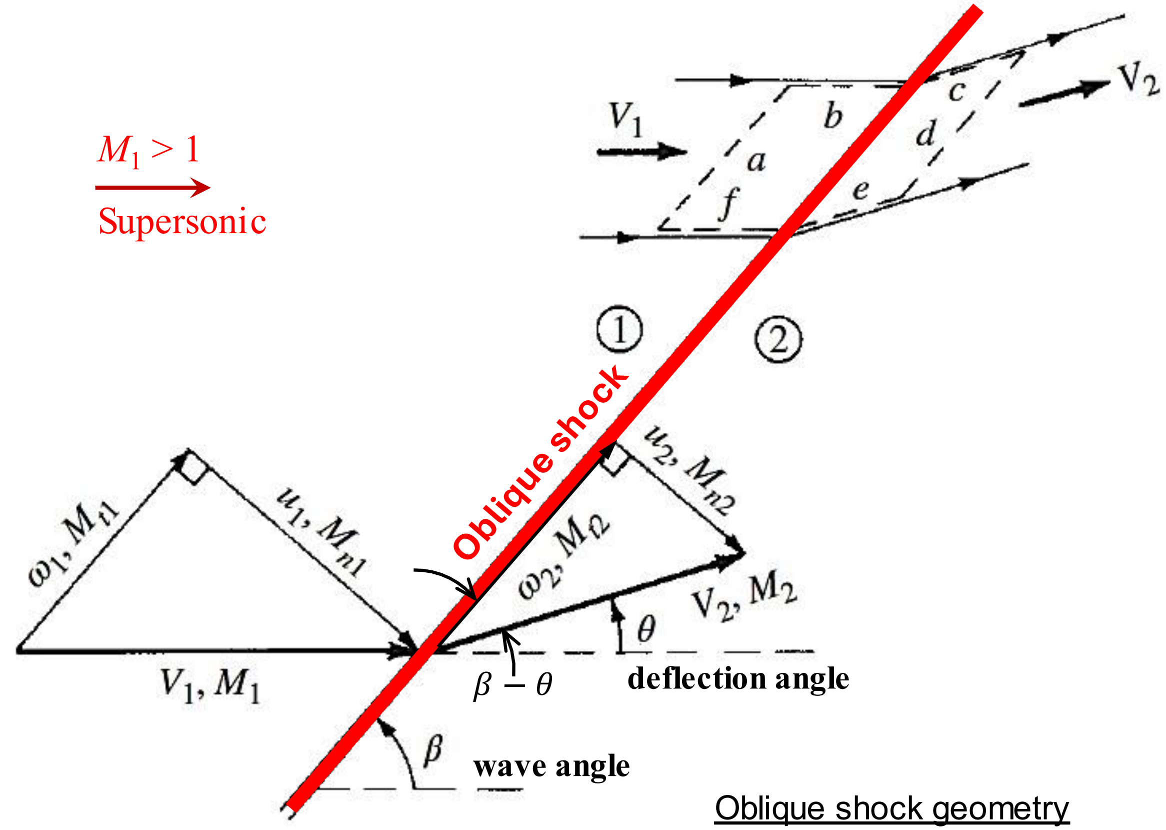
$$\Rightarrow \left(h_1 + \frac{V_1^2}{2}\right)\rho_1 u_1 = \left(h_2 + \frac{V_2^2}{2}\right)\rho_2 u_2 \qquad (\because h = e + pu)$$

Oblique Shock Waves

$$\Rightarrow h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \quad ; \left(\because \rho_1 u_1 = \rho_2 u_2\right)$$

$$\Rightarrow h_1 + \frac{u_1^2 + w_1^2}{2} = h_2 + \frac{u_2^2 + w_2^2}{2}$$

$$\Rightarrow h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \quad ; \left(\because w_1 = w_2\right)$$



Oblique shock geometry

So, governing equations for oblique shock wave are:

$$\rho_1 u_1 = \rho_2 u_2$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

These are identical in form to the normal shock continuity, momentum, and energy equations.

In both sets of equations, the **velocities are normal to the wave**.

Therefore, the changes across an oblique shock wave are governed by the normal component of free-stream velocity.

Oblique Shock Waves

For an oblique shock wave-

$$M_{n1} = M_1 \sin\beta$$

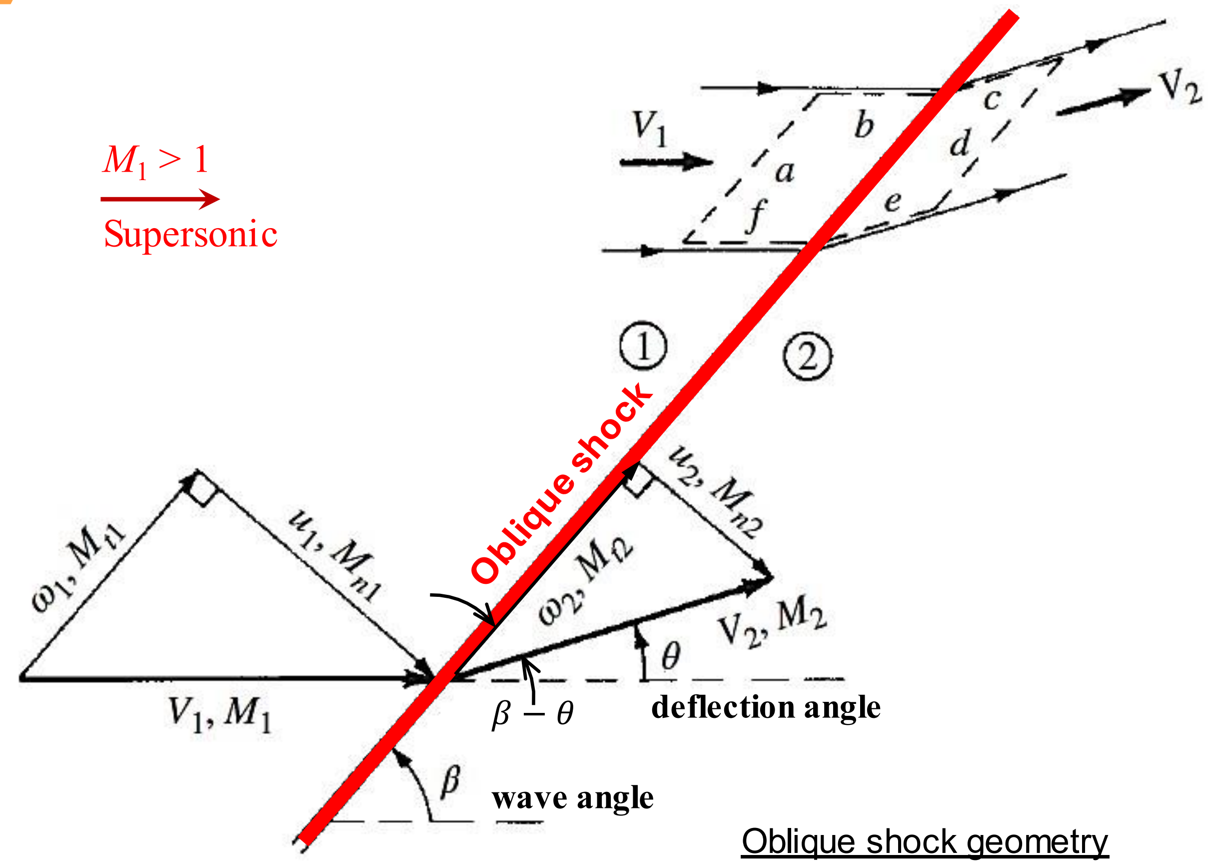
Identical expressions for changes across an oblique shock in terms of the normal component of the upstream Mach number can be obtained as **in case of normal shock wave**. Thus, for a calorically perfect gas;

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_{n1}^2}{(\gamma-1)M_{n1}^2+2}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_{n1}^2-1)$$

$$M_{n2}^2 = \frac{M_{n1}^2 + [2/(\gamma-1)]}{[2\gamma/(\gamma-1)]M_{n1}^2 - 1}$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2}$$



Oblique shock geometry

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2-1)$$

Normal shock relations

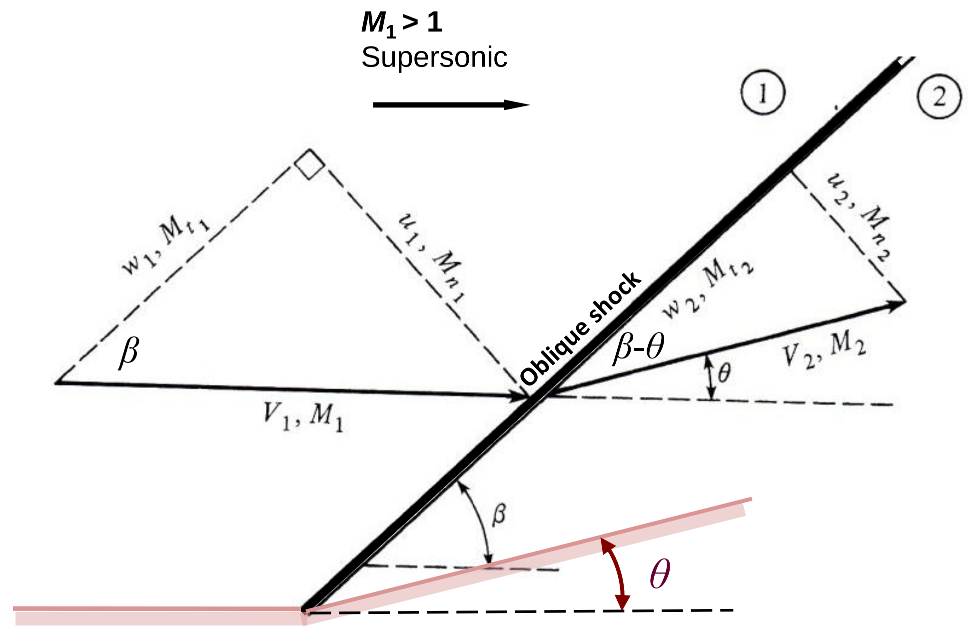
Oblique Shock Waves

Mach number behind the oblique shock, M_2

$$\sin(\beta-\theta)=\frac{M_{n2}}{M_2}$$

$$\Rightarrow M_2=\frac{M_{n2}}{\sin(\beta-\theta)}$$

M_{n2} is known from previous expression



The changes across an oblique shock are a function of two quantities: M_1 and β.

In reality, normal shocks are just a special case of oblique shocks where-

$$\beta=\frac{\pi}{2}$$

Oblique Shock Waves

M_2 cannot be found until the flow deflection angle θ is obtained. However, θ is also a unique function of M_1 and β

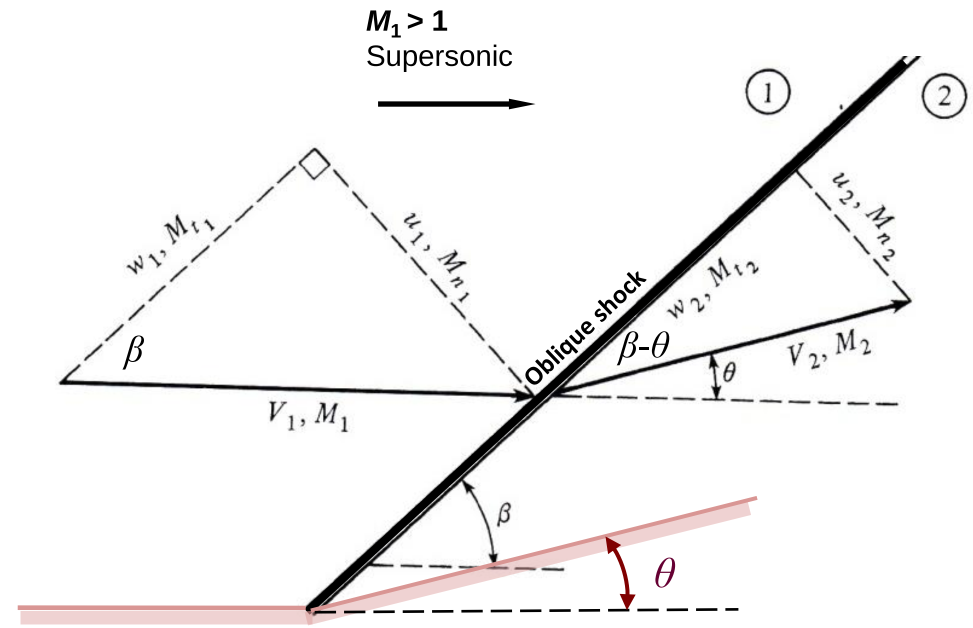
$$\tan\beta = \frac{u_1}{w_1}$$

and $$\tan(\beta-\theta) = \frac{u_2}{w_2}$$

Noting that $w_1 = w_2$

$$\therefore \quad \frac{\tan(\beta-\theta)}{\tan\beta} = \frac{u_2}{u_1}$$

$$\Rightarrow \frac{\tan(\beta-\theta)}{\tan\beta} = \frac{2+(\gamma-1)M_1^2\sin^2\beta}{(\gamma+1)M_1^2\sin^2\beta}$$



Using these relations

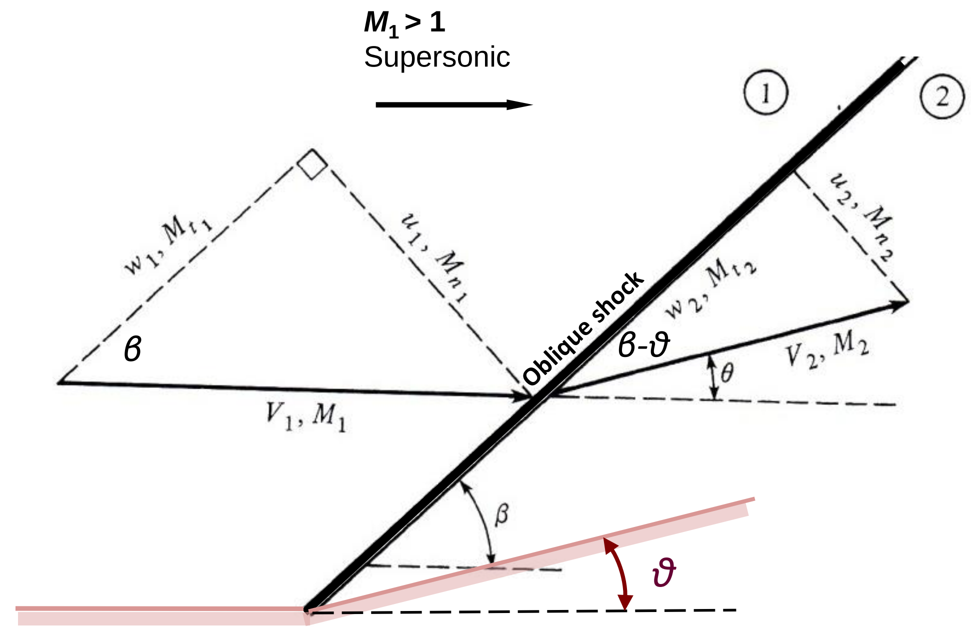
$$\begin{cases} \rho_1 u_1 = \rho_2 u_2 \\ M_{n1} = M_1 \sin\beta \\ \dfrac{\rho_2}{\rho_1} = \dfrac{(\gamma+1)M_{n1}^2}{(\gamma-1)M_{n1}^2+2} \end{cases}$$

Oblique Shock Waves

With some trigonometric manipulation, this equation can be expressed as-

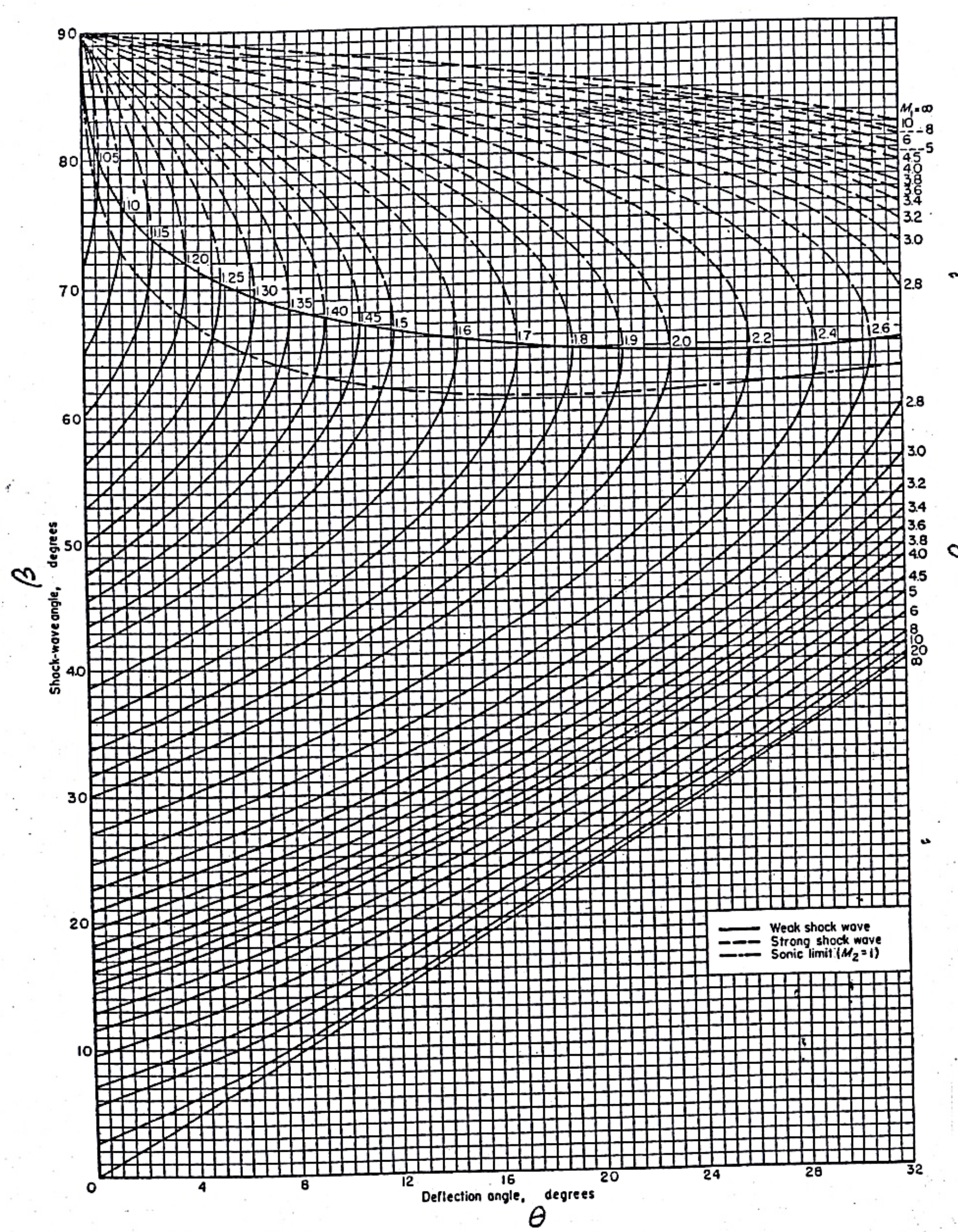
$$\Rightarrow \tan\theta = 2\cot\beta\left[\frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}\right]$$

This is well know $\theta - \beta - M$ relation

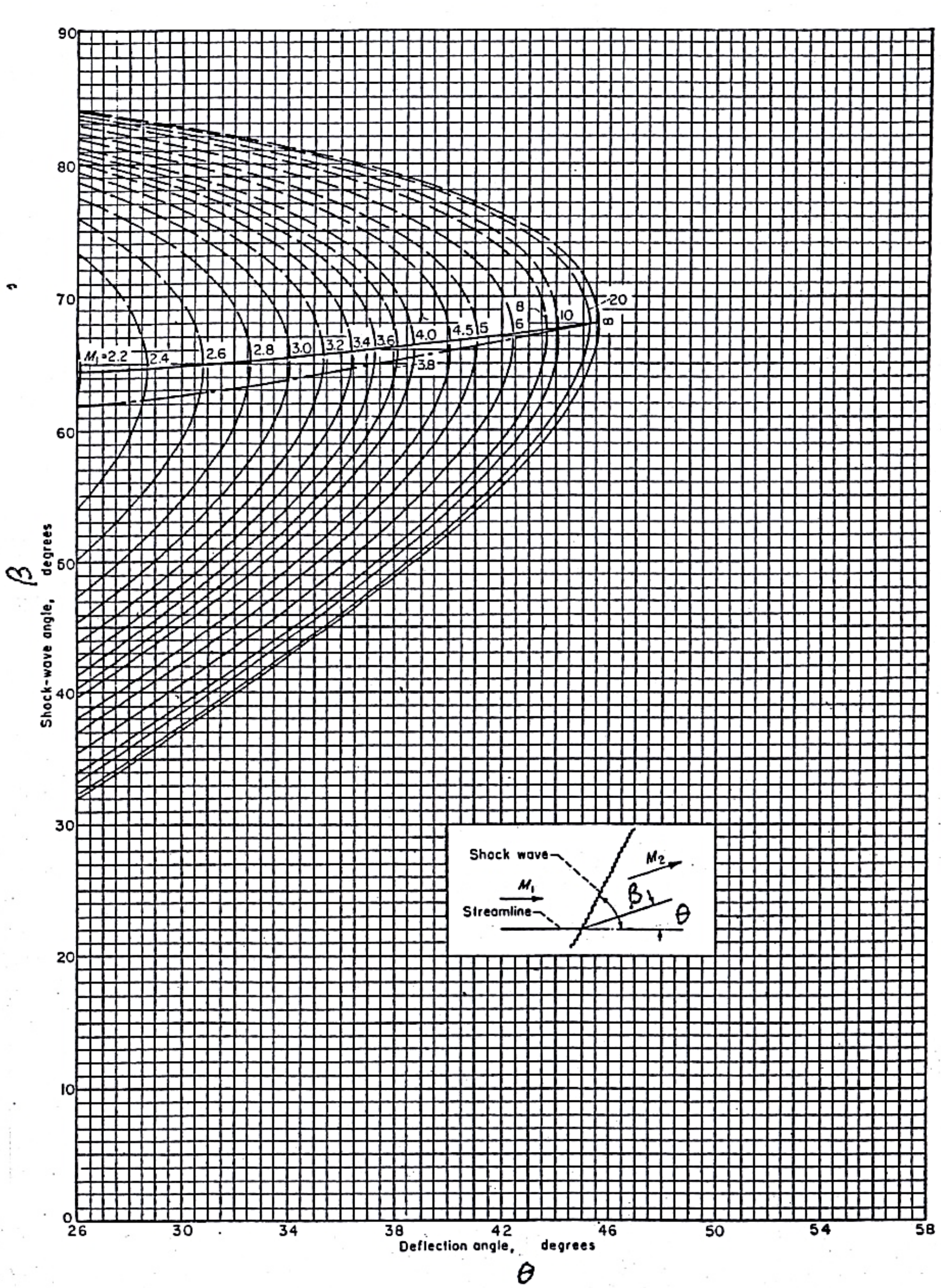


Oblique Shock Waves

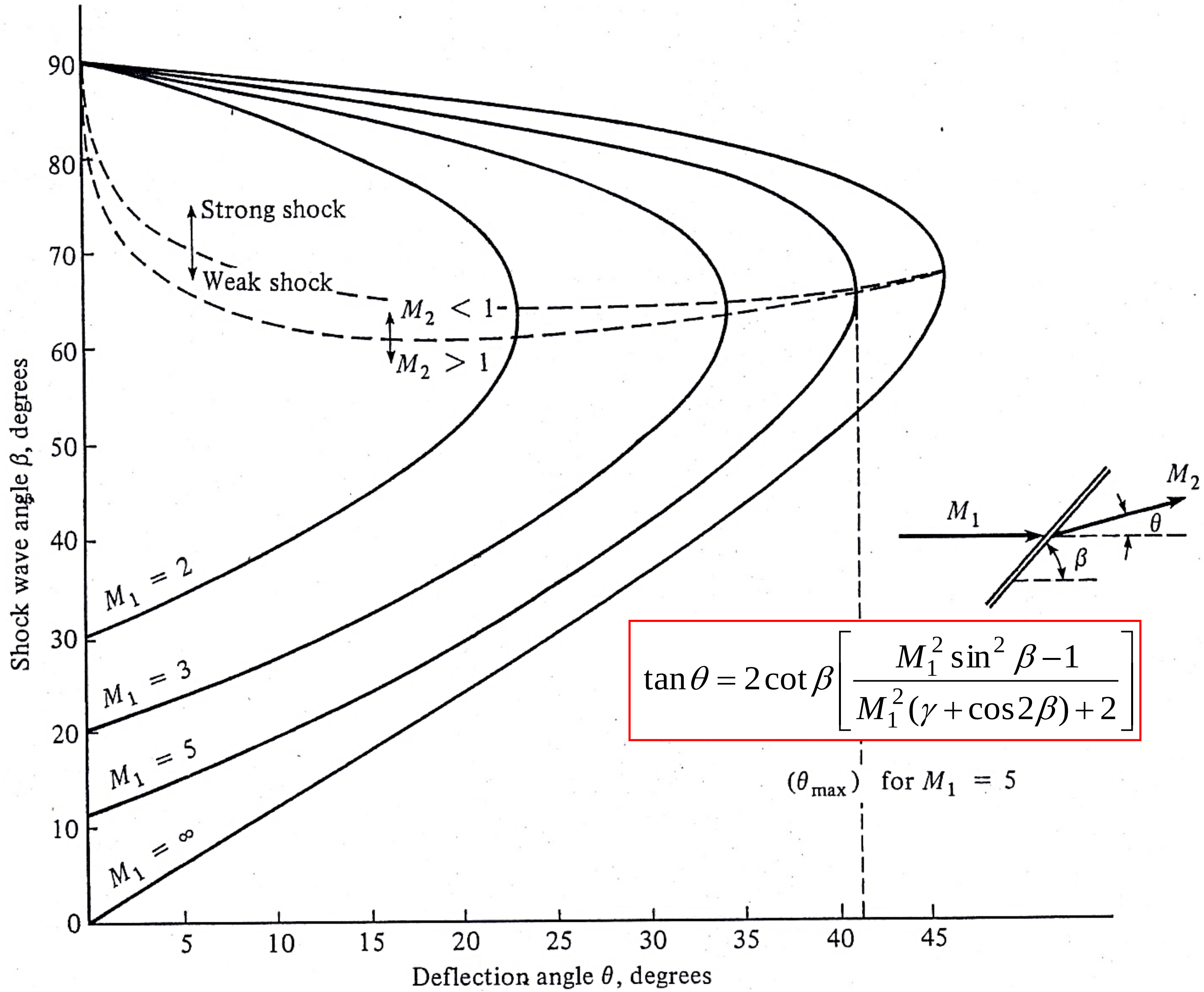
OBLIQUE SHOCK PROPERTIES: $\gamma = 1.4$



OBLIQUE SHOCK PROPERTIES: $\gamma = 1.4$



Oblique Shock Waves



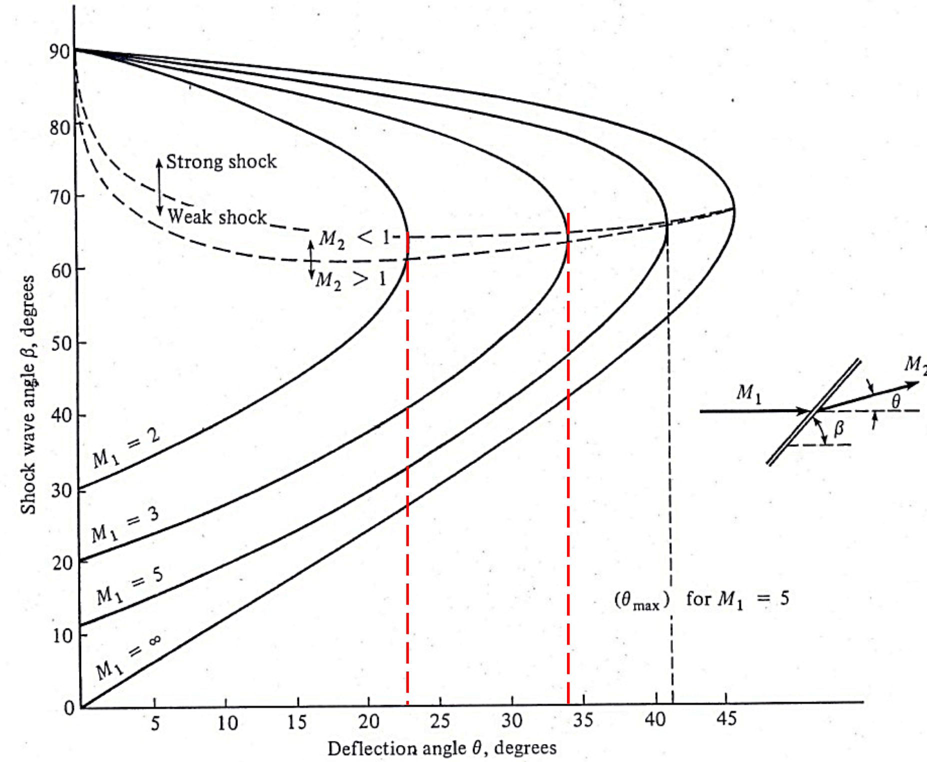
$$\tan\theta = 2\cot\beta\left[\frac{M_1^2 \sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}\right]$$

Figure 4.8 | *θ-β-M* curves. Oblique shock properties. *Important:* See front end pages for a more detailed chart.

Oblique Shock Waves

1. For any given M_1, there is a maximum deflection angle θ_{max}.

   for example, in case of $M_1 = 2$, $\theta_{max} = 23°$; in case of $M_1 = 5$, $\theta_{max} = 41°$ and so on.



If the physical geometry is such that $\theta > \theta_{max}$, then no solution exists for a straight oblique shock wave.

Instead, the shock will be curved and detached, as shown in figure.

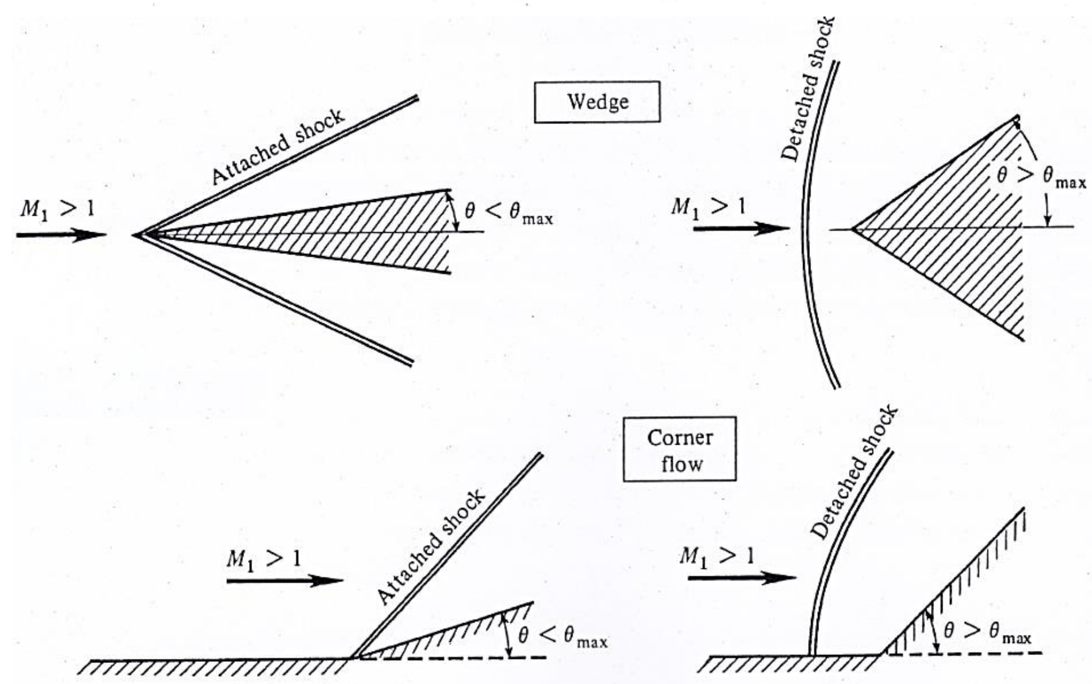
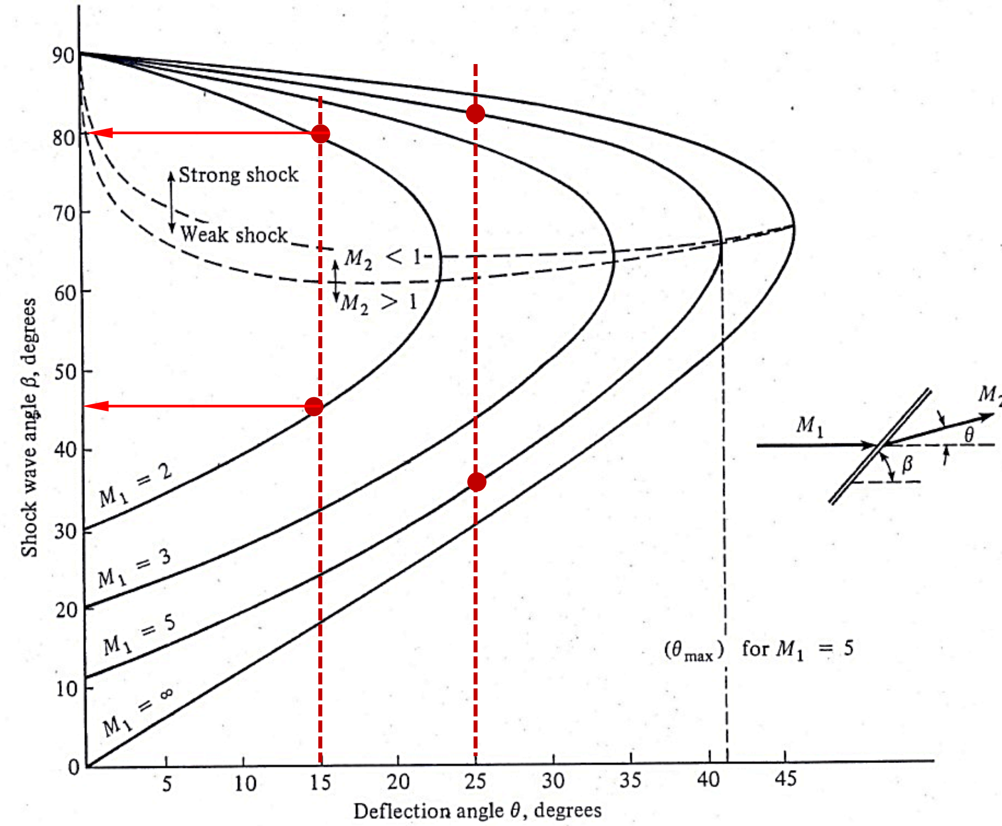


Figure 4.9 | Attached and detached shocks.

2. For any given $\theta < \theta_{max}$ (attached and straight shock), **there are two values of *β* predicted** by *θ-β-M* relation for a given Mach number.

   For example, in case of M_1= 2, and θ = 15°; two values of *β* are possible and these are 43° and 80°.

   Large value of *β* is called strong shock solution and small value of *β* is called the weak shock solution.



In nature, the weak shock solution is favored, and usually occurs. However, whether the weak or strong shock solution occurs is determined by the backpressure.

In <u>strong shock solution</u>, M_2 is subsonic (M < 1).
In <u>weak shock solution</u>, M_2 is supersonic (M > 1).

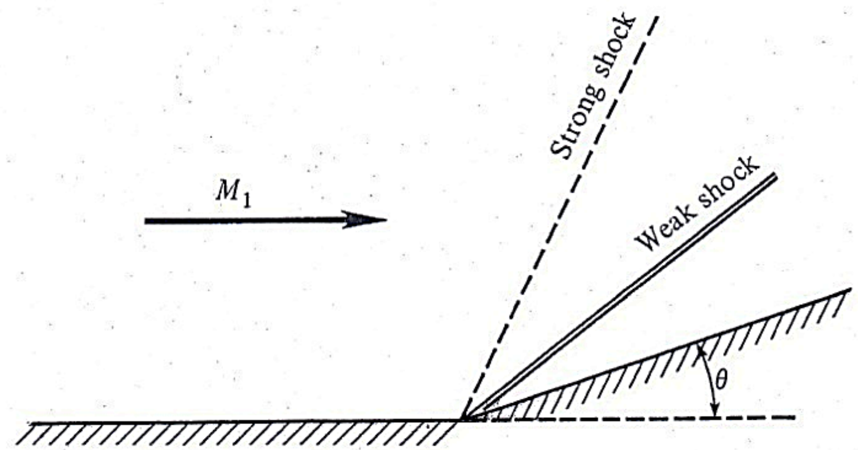


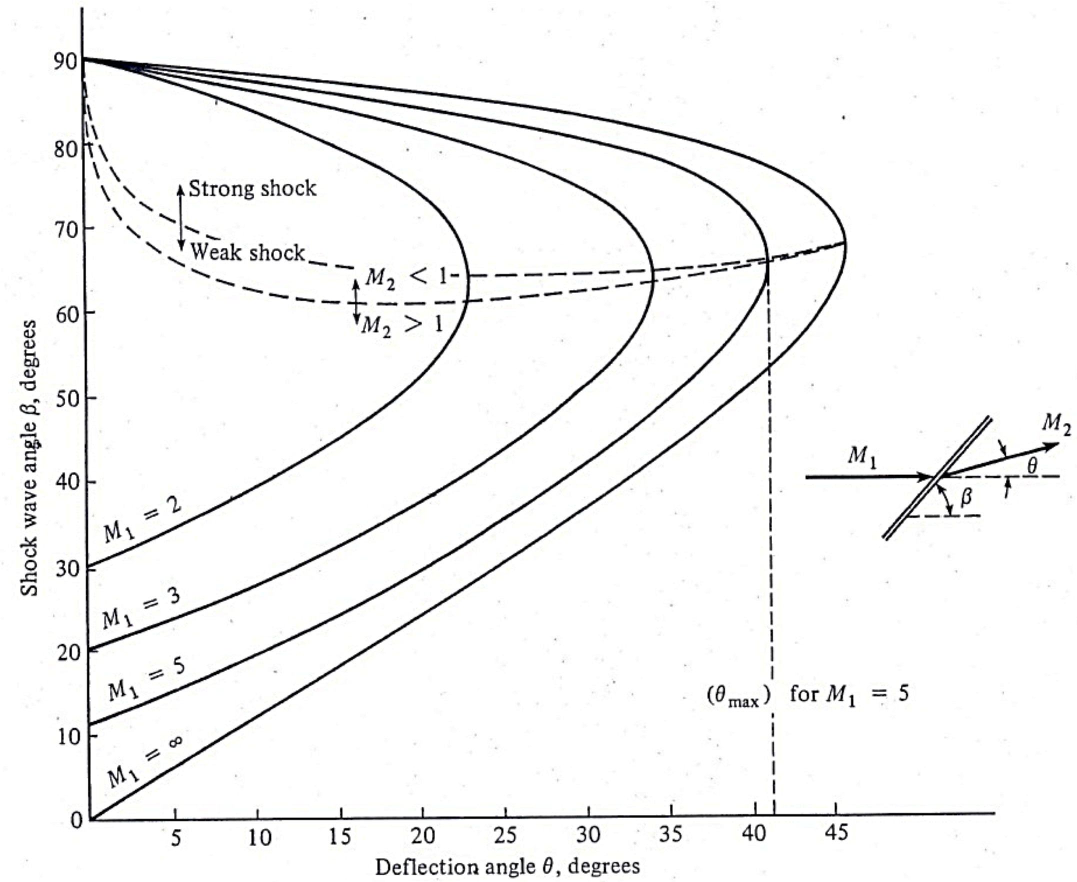
Figure 4.10 | Weak and strong shocks.

*** In case of oblique shock, M_1 >1 but M_2 could be supersonic (M > 1) or subsonic (M < 1)**

(difference from the normal shock wave)

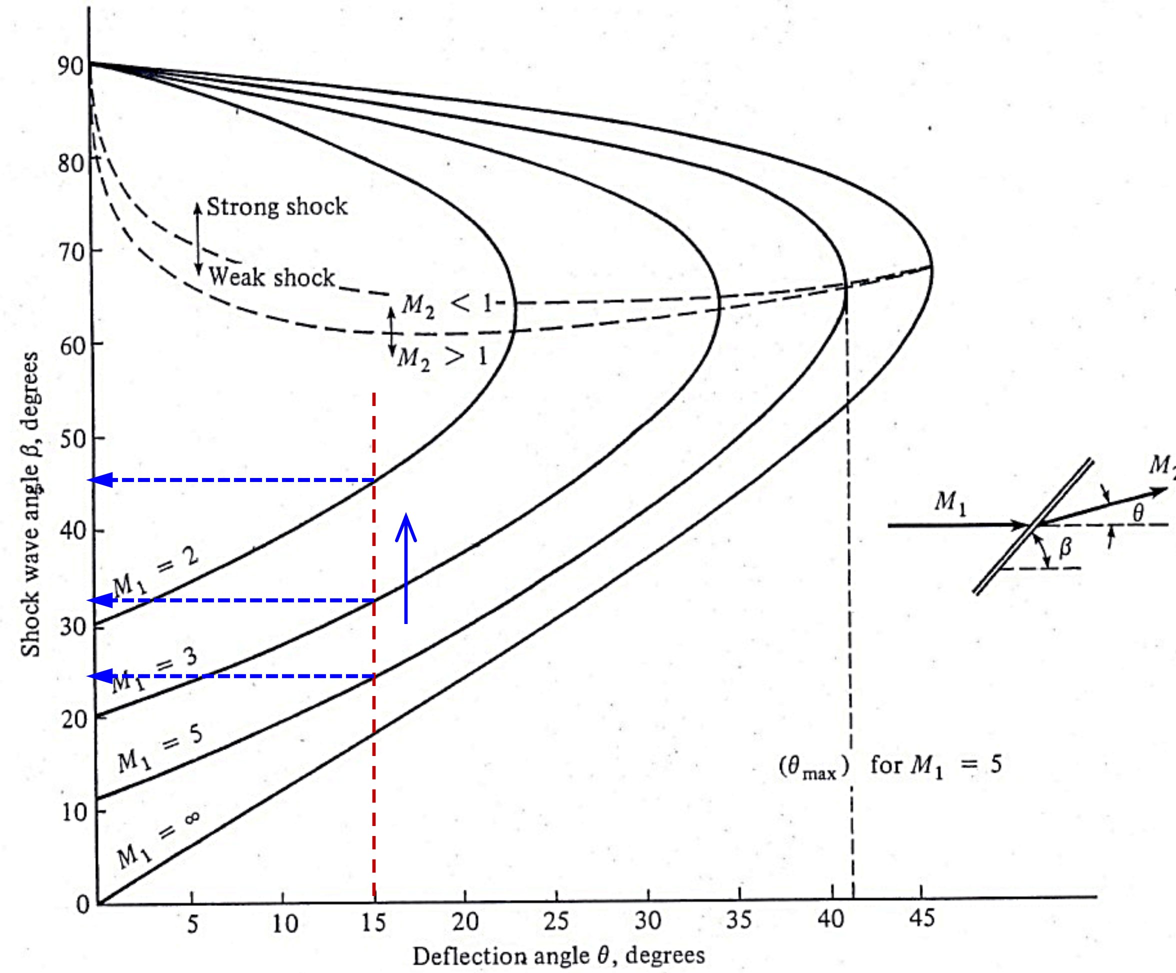
3. If $\theta = 0°$, then $\beta = 90°$ corresponding to a **normal shock** or $\beta = \mu$ corresponding to a **Mach wave** (shock wave with infinitesimal strength $M_1 = 1$ and $M_2 = 1$).

For example, in case of $M_1 = 2$, $\theta = 0°$; Mach wave angle is $\beta = \mu = 30°$.



Oblique Shock Waves

4. For a fixed deflection angle θ, as the free-stream Mach number decreases from high to low supersonic values, the wave angle increases (for weak shock solution).



Problem

(Example 9.2 Anderson)

Consider a supersonic flow with $M = 2$, $p = 1$ atm, and $T = 288$ K. This flow is deflected at a compression corner through 20°. Calculate M, p, T, p_0, and T_0 behind the resulting oblique shock wave.

Problem

(Example 9.5 Anderson)

Consider a Mach 3 flow. It is desired to slow this flow to a subsonic speed. Consider two separate ways of achieving this: (1) the Mach 3 flow is slowed by passing directly through a normal shock wave; (2) the Mach 3 flow first passes through an oblique shock with a 40° wave angle, and then subsequently through a normal shock. These two cases are sketched in Figure 9.14.

Calculate the ratio of the final total pressure values for the two cases, that is, the total pressure behind the normal shock for case 2 divided by the total pressure behind the normal shock for case 1. Comment on the significance of the result.

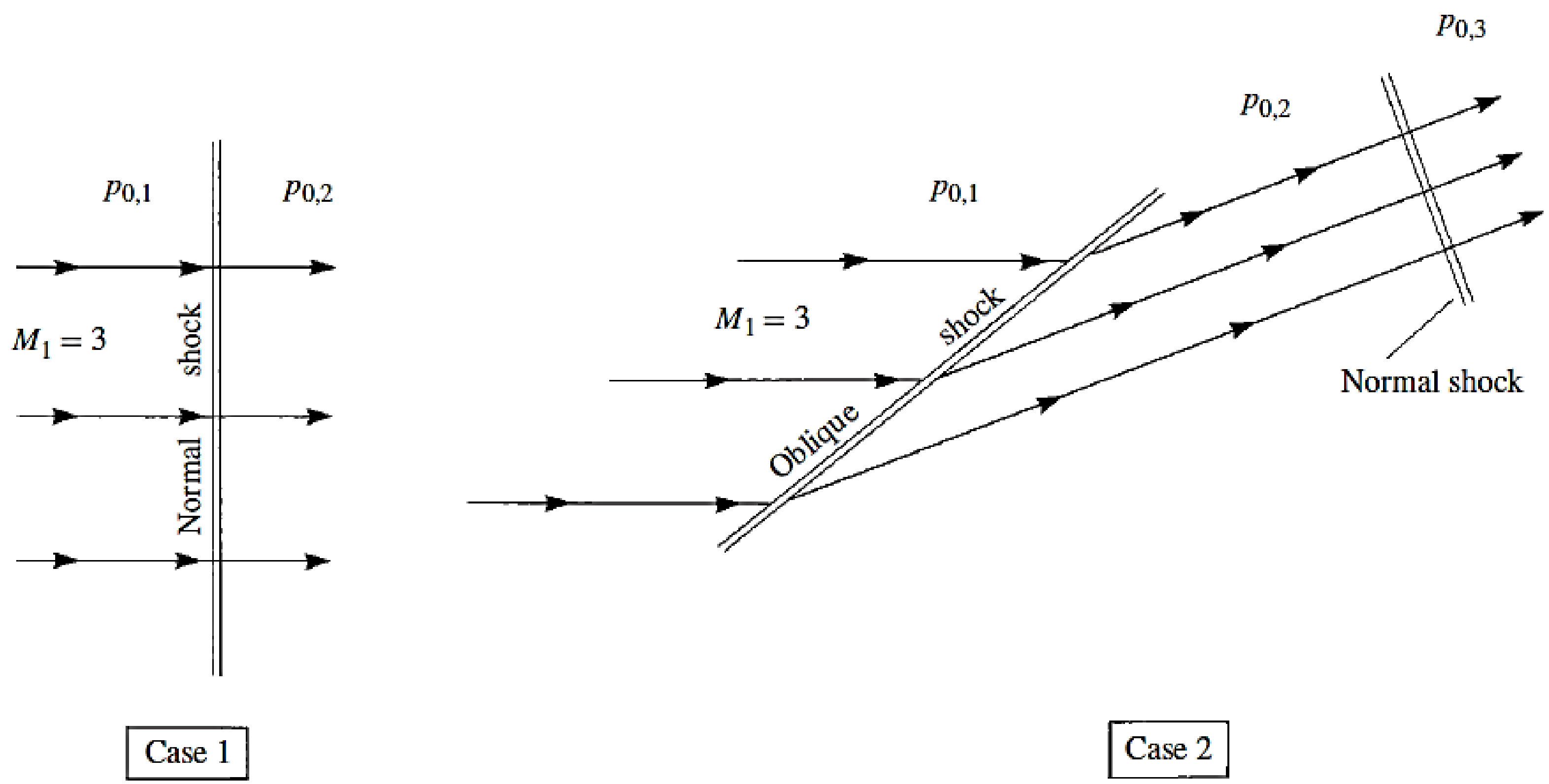
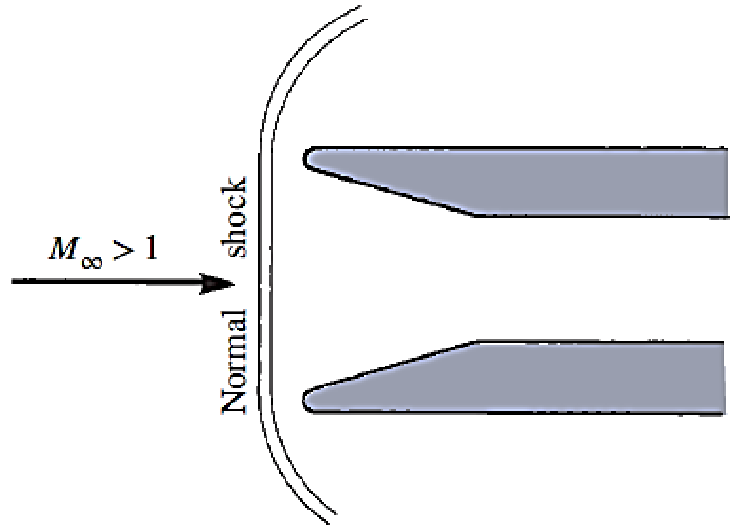
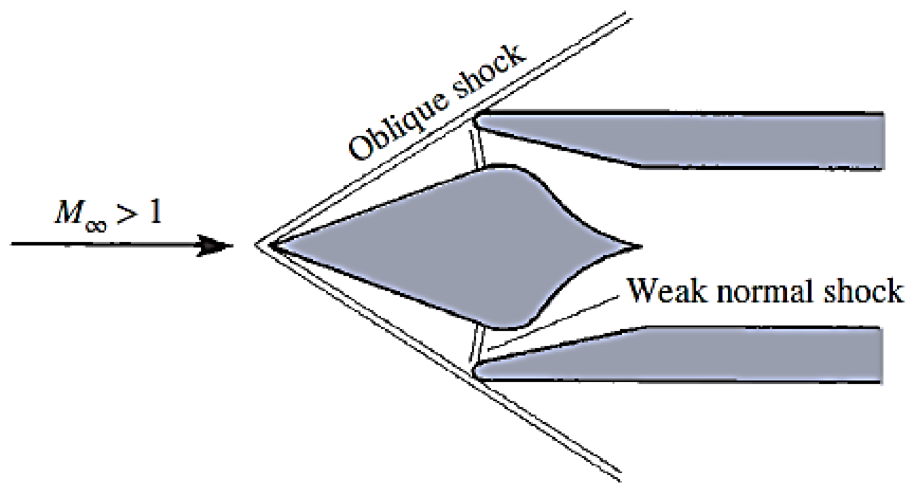


Figure 9.14 Illustration for Example 9.4.

Practical application in Scramjet Intake



(*a*) Normal shock inlet



(*b*) Oblique shock inlet

Figure 9.15 Illustration of (*a*) normal shock inlet and (*b*) oblique shock inlet.